

RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College under University of Calcutta)

B.A./B.SC. FOURTH SEMESTER EXAMINATION, MAY 2012

SECOND YEAR

MATHEMATICS (General)

Date : 29/05/2012

Time : 11 am – 2 pm

Paper : IV

Full Marks : 75

[Use separate Answer Books for each group]

Group – A

(Answer any four)

1. a) Examine the convergence of $\int_0^{\infty} \frac{dx}{(x+1)(x+2)}$. [3]
b) Show that the integral $\int_0^1 \frac{dx}{1-x}$ diverges. [2]
2. a) Evaluate : $\int_0^{\infty} e^{-x^2} dx$. [2]
b) Show that the integral : $\int_0^1 \frac{dx}{(1-x^6)^{1/6}} = \frac{\pi}{3}$. [3]
3. Evaluate $\iint xy(x+y)dx dy$ over the area bounded by $y = x^2$ and $y = x$. [5]
4. Find the perimeter of the Cardioide $r = a(1 - \cos \theta)$. Show that the arc of the upper half of the curve is bisected at $\theta = \frac{2\pi}{3}$. [3+2]
5. Find the volume of the solid obtained by revolving the cycloid $x = a(\theta + \sin \theta)$, $y = a(1 + \cos \theta)$ about its base. [5]
6. Show that $\int_0^{\pi/2} \frac{d\theta}{\sqrt{\cos \theta}} \times \int_0^{\pi/2} \sqrt{\cos \theta} d\theta = \pi$. [5]

Or,

Using double integral find the area bounded by the curve $y^2 = x^3$ and the line $y = 2x$. [5]

Group – B

Unit – I

(Answer any two)

7. a) Find the orthogonal trajectories of the family of curves $r = a(1 + \cos \theta)$. [3]
b) Find the particular integral of the differential equation $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 5y = 10\sin x$. [2]
8. Solve : $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = e^{2x}$ given that $y = 0$ and $\frac{dy}{dx} = 1$ when $x = 0$. [5]
9. Find the complete primitive of the differential equation $\frac{d^2y}{dx^2} + \frac{1}{x} \frac{dy}{dx} = \frac{12 \log x}{x^2}$. [5]

Unit - II

(Answer any five)

10. a) State analytically when two events are called mutually exclusive. If A, B, C, be three events such that $P(B) = \frac{3}{2}\{P(A)\}$, $P(C) = \frac{1}{2}\{P(B)\}$ find $P(A)$ given that A, B, C are mutually exclusive and exhaustive events in the sample space. Define random experiment. [1+2+2]

- b) A random variable X has the probability density function $f(x) = Kx(2-x)$ $0 \leq x \leq 2$ find K and $\text{Var}(X)$. [1+3]
11. a) A problem is given to three student whose chances of solving it are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$. What is the chance that the problem will be solved? [5]
- b) The probability that a man hits a target is $\frac{1}{3}$. How many times must he fire so that the probability of hitting the target at least once is more than 90%. [4]
12. a) State and prove Baye's theorem. [1+4]
- b) There are two identical urns containing respectively 5 white and 3 red balls and 4 white and 6 red balls. An urn is chosen at random and a ball is drawn from it. Find the probability that the ball is white. [4]
13. a) Find the mean and variance of the Poisson (μ) distribution. [4]
- b) The random variables X and Y have the joint density function
- $$f(x, y) = \begin{cases} 6(1-x-y) & , \text{ for } x > 0, y > 0, x+y < 1 \\ 0 & , \text{ elsewhere} \end{cases}$$
- Find the marginal distributions of x and y . Are X and Y independent? [5]
14. a) Calculate Fisher's price index number for the year 2003 with the year 2000 as base, from the following data : [4]

Commodity	2000		2003	
	Price	Quantity	Price	Quantity
A	8	80	12	140
B	12	105	15	150
C	15	150	20	200

- b) The median and mode of the following frequency distribution are respectively known as 27 and 26. Find the values of a and b . [5]
- | | | | | | |
|-----------|--------|---------|---------|---------|---------|
| Value | 0 – 10 | 10 – 20 | 20 – 30 | 30 – 40 | 40 – 50 |
| Frequency | 3 | a | 20 | 12 | b |
15. a) Fit a straight line trend for the following series and estimate the value for 1972 : [5]
- | | | | | | | | |
|------------|--------|------|------|------|------|------|------|
| Year | : 1965 | 1966 | 1967 | 1968 | 1969 | 1970 | 1971 |
| Production | : 125 | 128 | 133 | 135 | 140 | 141 | 143 |
- b) Write down the probability density function of "Snedecor's F-distribution". Define with example of "Point Estimation". [4]
16. a) Construct a frequency distribution table to draw the histogram of the following data. 96, 130, 63, 115, 145, 99, 118, 104, 126, 72, 77, 87, 151, 81, 142, 122, 110, 131, 98, 96. [5]
- b) The marks obtained by 17 candidates in an examination have a mean 57 and variance 64. Find 99% confidence interval for the mean of the population of marks, assuming it to be normal. [4]
- (Given that $t_E = 2.921$)
17. Find out the skewness and kurtosis of the series by the method of moments : [5+4]
- | | | | | |
|-------------|----------|---------|---------|---------|
| Measurement | : 0 – 10 | 10 – 20 | 20 – 30 | 30 – 40 |
| Frequency | : 1 | 3 | 4 | 2 |

